Definition of limit of a function:

Let $f(x)$ be a function which is not defined at $x=a$ or you can say undefined at $x=a$, then the real number l is said to be limit of $f(x)$ at $x=a$, if for an arbitrary small positive value $\epsilon>0$, $\exists$ a positive number $\delta>0$ s.t.

$$|f(x)-l|<\epsilon, \text{ when } 0<|x-a|<\delta$$

Symbolically, we write it as

$$\lim_{x\to a} f(x) = l.$$

Here it should be clear that $\lim_{x\to a}$ means that x tends to a or x approaches a. that again means, the variable x takes the value very close to a but not exactly equal to a.

$$|x-a|<\delta \Rightarrow a-\delta<x<a+\delta \Rightarrow x\in(a-\delta, a+\delta).$$

LHL: Left hand limit:- when x approaches from left of a, then limit is called left hand limit

$$\Rightarrow f(a-) = \lim_{x\to a-} f(x)$$

RHL: Right hand limit: when x approaches from right of a, then limit is called right hand limit. $\Rightarrow f(a+) = \lim_{x\to a+} f(x)$

If LHL = RHL at $x = a$ then it is called limit of the function at $x = a$

$$\Rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x) = l.$$

Infinite limit $\Rightarrow \lim_{x \to a} f(x) = +\infty$

$\Rightarrow$ Let $f(x)$ be a function defined in deleted neighbourhood (n.b.d.) of a, then ~~f(x)~~ limit of $f(x)$ will be $+\infty$ as x tends to a if for every $K > 0$, however large, $\exists\ \delta > 0$

s.t. $f(x) > K$ whenever $0 < |x - a| < \delta$

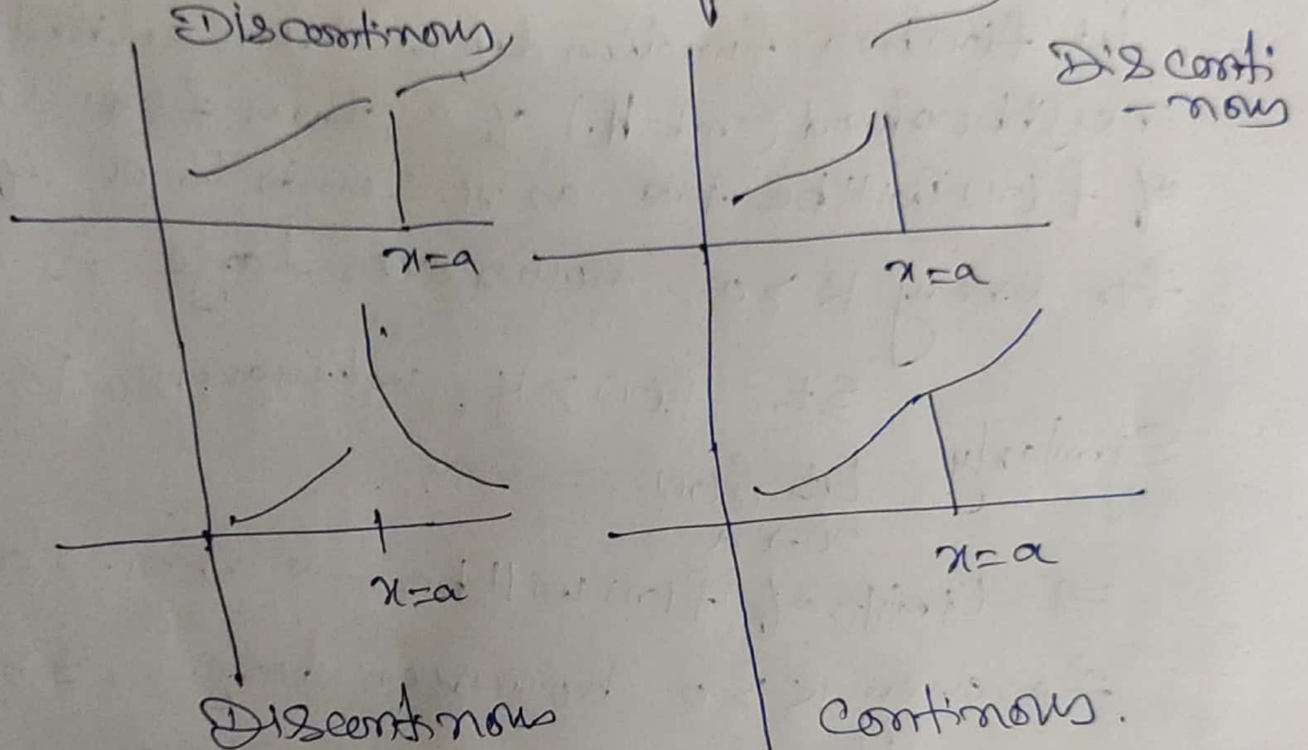
Similarly $\lim_{x \to a} f(x) = -\infty$

$\Rightarrow$ Limit of $f(x)$ will be $-\infty$ as $x \to a$ if for every $K > 0$, however large, $\exists\ \delta > 0$

s.t. $f(x) < -K$, whenever $0 < |x - a| < \delta$

Similarly If $\lim_{x \to \infty} f(x) = +\infty$ then it is also called infinite limit.

continuity :- If there is no brek or jump in the graph of the function in the given interval then the function is called continous in the interval. But this definition is non-mathematical because we can not draw the graph of all functions in a simple way, so we need a mathematical definition.



Cauchy's definition of continuity

A function $f(x)$ is said to be continous at $x = a \in D$ (domain D) if for any $\epsilon > 0$ (however small), $\exists\ \delta > 0$ (depends on ϵ) s.t.

$$|f(x) - f(a)| < \epsilon \quad \text{when} \quad |x-a| < \delta$$

or we can say that $f(x)$ is said to be continous at $x=a$, if for every $\epsilon > 0$, $\exists$ an interval $(a-\delta, a+\delta)$ s.t. for every $x \in (a-\delta, a+\delta)$, the difference of $f(x)$ and $f(a)$ can be made as small as we want.

$x \to a+$